UNSTEADY MOTION OF A THIN FOIL IN AN INCOMPRESSIBLE FLUID

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(NASA-TT-F-16300) UNSTEADY MOTION OF A THIN FOIL IN AN INCOMPRESSIBLE FLUID (Kanner (Leo) Associates) 12 p HC $3.25 CSCL 20D
N75-22623
Unclas
G3/34 19491

Translation of "Asüpra miscarii nestationare a unui profil subtire intr-un fluid incompresibil," Studii i Cercetari Matematice, Vol. 26, No. 4, 1974, pp. 537-544

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 MAY 1975

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TT F-16,300		
4. Title and Subtitle UNSTEADY MOTION OF A THIN FOIL IN AN INCOMPRESSIBLE FLUID		5. Report Date May 1975
		6. Performing Organization Code
7. Author(s) D. Homentcovschi		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		11. Contract or Grant No. NASw-2481
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Asupra miscarii nestationare a unui profil subtire intr-un fluid incompresibil," Studii i Cercetari Matematice, Vol. 26, No. 4, 1974, pp. 537-544

16. Abstract Consideration of the unsteady motion of an incompressible viscous fluid past a thin foil, using the hydrodynamic equations written in terms of distributions as the point of departure. It is shown that such a procedure makes it possible to avoid reliance on Birnbaum's hypothesis regarding the continuous separation of vortices at the trailing edge of the foil. Instead, a small-perturbation hypothesis is made, which makes it possible to linearize the equations of motion and the boundary conditions and to use these on the projection of the foil on the x-axis.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	10	

NASA-HQ

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1. Introduction /537*

The study of the motion of a thin airfoil in a nonviscous, incompressible fluid was the object of numerous investigations (for a brief bibliography on this field, consult the textbook of fluid mechanics by Academician Caius Jacob [1]). In order to explain the variation in circulation time of the fluid velocity about the airfoil, all the authors base themselves on W. Birmbaum's hypothesis, according to which there is a continuous formation of vortices at the trailing edge of the airfoil. This implies the existence of a discontinuity line of velocities in that part of the Ox axis which the airfoil crosses in its motion.

We shall attempt a slightly different approach to the problem. We shall use the equations of fluid mechanics written in terms of distributions (2), which will allow us to avoid reliance on the hypothesis dealing with the existence and the shape of the discontinuity surface. The small-perturbation hypothesis used by us enables us to linearize the equations of motion, the limiting conditions, and permits us to use the latter in the projection of the airfoil on the Ox axis. Let us note that this hypothesis was used in previous studies dealing with this problem, since otherwise the jump conditions resulting from the theorem of conservation of the momentum would not be verified on the discontinuity surface.

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* Numbers in the margin indicate pagination in the foreign text.

Subsequently, a representation is obtained for the velocity field which has all the characteristics of the previously determined solution to this problem. Thus, the existence of the discontinuity line of velocities on the segment (1 - s(t), 1) is a result of the established formula and its posulation is no /538
longer necessary. Consequently, W. Birnbaum's model is the only possible model for the unsteady motion of an incompressible fluid.

The general case of the airfoil with finite thickness is also discussed in this study. If the area of the airfoil changes in time, it is shown that there is a source line superposed on the voltage line on segment (1 - s(t), 1) of the axis Ox.

2. Formulation of the Problem

Let us consider a thin airfoil profile whose equations are

$$y_1 = \varepsilon \tilde{Y}_{\pm}(x_1; t), \qquad -1 < x_1 < 1, \quad t > 0 \tag{1}$$

with regard to a trihedron bound to the airfoil, which moves with a velocity $\vec{V}_e(t)$ in an incompressible fluid which is nonviscous and fills the whole plane. Let us admit that the direction of axis O_1x_1 coincides with the direction of the airfoil speed, while its sense is opposite to the sense of the vector. The unsteady nature of the motion is due both to the fact that the vector $\vec{V}_e(t)$ has a time-variable modulus, and due to vibrations of the airfoil given by Eq. (1). We shall relate the motion of the gluid to a fixed trihedron Oxyz which is none other than the position t = 0 of the $O_1x_1y_1z_1$ trihedron. We thus obtain

$$\begin{aligned} x_1 &= x + \int_0^t V_e(\theta)\, d\theta \equiv x + \sigma(t), \\ y_1 &= y. \end{aligned} \tag{2}$$

The equations in terms of distribution characterizing the motion of the fluid have the following form:

$$\operatorname{div}\vec{v} = 0, \tag{3}$$

$$\frac{\partial \vec{v}}{\partial t} + \operatorname{grad} p = \varepsilon l(x;t)\,\theta(x;t)\,\delta(y)\vec{i}_1 + n(x;t)\theta(x,t)\,\delta(y)\,\vec{i}_2 \tag{4}$$

In these equations $\vec{v}(x,y,t)$ and $p(x,y,t)$ denote the perturbations of velocity and pressure, respectively, $\varepsilon l(x;t)$ and $\varepsilon n(x;t)$ are magnitudes which characterize the resistance to forward motion and the lifting force which acts on the airfoil, and

$$\theta(x,t) = \begin{cases} 1 & \text{for } -1-s < x < 1-s \\ 0 & \text{for } x \in (-\infty; -1-s) \cup (1-s, +\infty) \end{cases}$$

is a function which is characteristic for the projection of the airfoil profile on the Ox axis. We shall write the limiting conditions of the problem in a linearized form

$$\left.\begin{aligned} v_y(x; +0; t) &= \varepsilon Y_+(x; t), \\ v_y(x; -0; t) &= \varepsilon \tilde{Y}_-(x; t), \end{aligned}\right\} \quad x \in (-1-s, 1-s) \tag{5}$$

where $y = \varepsilon Y_{\pm}(x_1;t) \equiv \varepsilon \tilde{Y}_{\pm}(x + s;t)$ are the equations of the two /539
arches of the profile with regard to the system of fixed axes.

Applying the Laplace transform with regard to time and the Fourier transform with regard to space variables of systems (3) and (4), the following equations are obtained:

$$\begin{cases} ik_1 U(k_1;k_2;k) + ik_2 V(k_1;k_2;k) = 0, \\ kU(k_1;k_2;k) + ik_1 P(k_1;k_2;k) = \\ \qquad = \mathcal{F}\{\varepsilon l(x;t)\,\theta(x;t)\delta(y) + u(x;y;0)\}, \\ kV(k_1;k_2;k) + ik_2 P(k_1;k_2;k) = \\ \qquad = \mathcal{F}\{n(x,t)\theta(x,t)\,\delta(y) + v(x;y;0)\}, \end{cases} \tag{6}$$

where $U(k_1;k_2;k) = \int_0^\infty e^{-kt} \iint_{-\infty}^{+\infty} u(x;y;t)\, e^{-(ik_1x+ik_2y)}\, dx\, dy \equiv \mathcal{F}\{u(x;y;t)\}$ etc...

Hence we obtain

$$U(k_1;k_2;k) = [k(k_1^2+k_2^2)]^{-1}\,[k_2^2\mathcal{F}\{\varepsilon l(x;t)\,\theta(x;t)\,\delta(y) + u(x;y;0)\} -$$
$$- k_1k_2\mathcal{F}\{n(x;t)\,\theta(x;t)\,\delta(y) + v(x;y;0)\}],$$
$$V(k_1;k_2;k) = [k(k_1^2+k_2^2)]^{-1}\,[-k_1k_2\mathcal{F}\{\varepsilon l(x;t)\,\theta(x;t)\,\delta(y) + u(x;y;0)\} +$$
$$+ k_1^2\mathcal{F}\{n(x;t)\,\theta(x;t)\,\delta(y) + v(x;y;0)\}].$$

but since

$$\mathcal{F}^{-1}\{[k(k_1^2+k_2^2)]^{-1}\} = -\frac{\theta(t)}{4\pi}\ln(x^2+y^2)$$

we have

$$\begin{aligned} u(x;y;t) &= \frac{\theta(t)}{2\pi}\frac{x^2-y^2}{(x^2+y^2)^2} * [\varepsilon l(x;t)\,\theta(x;t)\,\delta(y) + u(x;y;0)] + \\ &+ \frac{\theta(t)}{2\pi}\frac{2xy}{(x^2+y^2)^2} * [n(x;t)\,\theta(x;t)\,\delta(y) + v(x;y;0)], \\ v(x;y;t) &= \frac{\theta(t)}{2\pi}\frac{2xy}{(x^2+y^2)^2} * [\varepsilon l(x;t)\,\theta(x;t)\,\delta(y) + u(x;y;0)] + \\ &+ \frac{\theta(t)}{2\pi}\frac{y^2-x^2}{(x^2+y^2)^2} * [n(x;t)\,\theta(x;t)\,\delta(y) + v(x;y;0)]. \end{aligned} \tag{7}$$

Or considering the complex velocity /540

$$w(x;y;t) = u(x;y;t) - iv(x;y;t) = (2\pi)^{-1}\,\theta(t)\,z^{-2} * \varepsilon l(x;t)\,\theta(x;t)\,\delta(y) +$$
$$+ i(2\pi)^{-1}\,\theta(t)\,z^{-2} * n(x;t)\,\theta(x;t)\,\delta(y) + i(2\pi)^{-1}\,\theta(t)\,z^{-2} *$$
$$* [u(x,y,0) + iv(x,y,0)]$$

where z = x + iy. The explicit expression of the complex velocity will be

$$w(x;y;t) = \frac{1}{2\pi}\int_0^t d\tau \int_{-1-s(\tau)}^{1-s(\tau)} \frac{c\,l(\xi,\tau)}{(z-\xi)^2}\,d\xi + \frac{i}{2\pi}\int_0^t d\tau \int_{-1-s(\tau)}^{1-s(\tau)} \frac{u(\xi,\tau)}{(z-\xi)^2}\,d\xi + \\ + \frac{1}{2\pi}\iint_{-\infty}^{+\infty} \frac{\bar{\omega}(\xi;\eta;0)}{(z-\xi-i\eta)^2}\,d\xi\,d\eta. \tag{8}$$

The last term of Eq. (8) can be seen immeddiately to be nothing else than v(x,y,0), so that we have

$$f(z;t) \equiv w(x;y;t) - w(x;y;0) = \\ = \frac{i}{2\pi}\int_0^t d\tau \int_{-1-s(\tau)}^{1-s(\tau)} \frac{u(\xi,\tau) - i c\,l(\xi,\tau)}{(z-\xi)^2}\,d\xi \tag{9}$$

3. Properties of the Function f(z;t)

1. The function f(z;t) is a olomorphic function of the variable v in the complex plane with section [-1 - s (t); +1] on the real axis. Indeed, function f(z;t) is expressed by a double integral which extends over the cross-hatched area in Fig. 1. By changing the order of integration we have:

$$f(z;t) = \frac{i}{2\pi}\int_{-1-s(t)}^{1} \frac{H(\xi,t)}{(z-\xi)^2}\,d\xi \tag{9'}$$

The expression of H(ξ,t) results from the integral with respect to τ. A function with the expression (9') is evidently olomorphic in the complex plane with section [-1 - s(t), 1].

2. The function f(z;t) behaves at large distances as z^{-2}. Thus, the condition of conservation of the circulation imposed by Thompson's theorem is fulfilled.

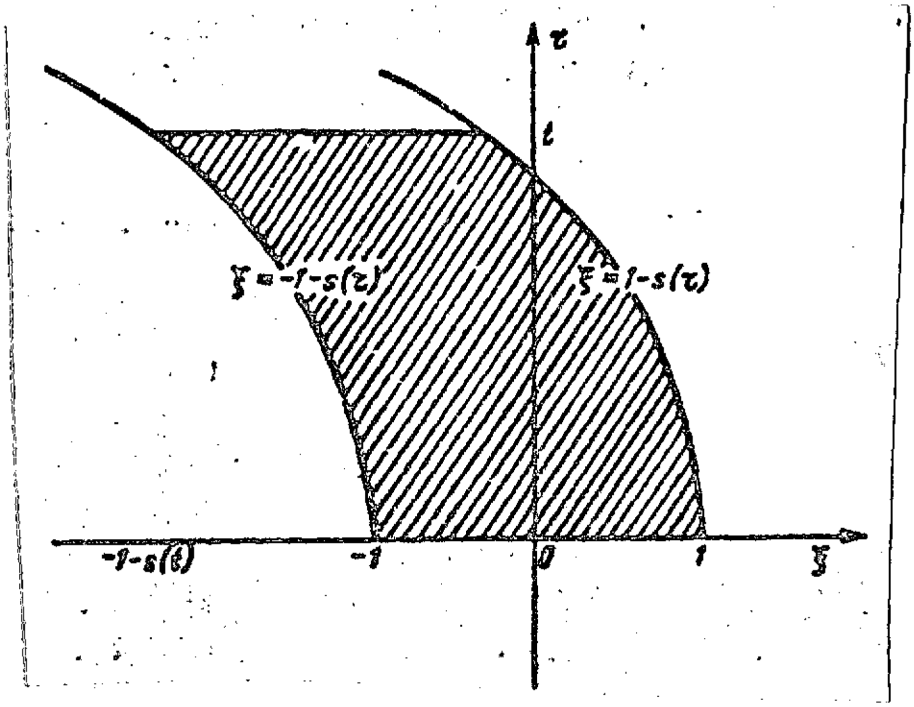

Fig. 1.

3. The derivative with respect to time of f(z;t) is an olomorphic function at the outside of the airfoil. Indeed we have

$$\frac{\partial f(z;t)}{\partial t} = \frac{i}{2\pi}\int_{-1-s(t)}^{1-s(t)} \frac{n(\xi,t) - i\varepsilon l(\xi,t)}{(z-\xi)^2}\, d\xi, \tag{10}$$

so that we now see that the above statement is evident.

4. Solution of the Problem /541

We shall use the limiting condition in order to determine the functions n(x;t) and l(x;t), which enter into formula (9). We shall write function f(z;t) in the form:

$$f(z;t) = \frac{\varepsilon}{2\pi}\int_0^t d\tau \int_{-1-s(\tau)}^{1-s(\tau)} \frac{l(\xi,\tau)}{(z-\xi)^2}\, d\xi + \tag{11}$$

$$+ \frac{i}{2\pi}\int_0^t d\tau \int_{-1-s(\tau)}^{1-s(\tau)} \frac{n(\xi,\tau)}{(z-\xi)^2}\, d\xi = f_1(z;t) + f_2(z;t).$$

On segments $(-\infty, -1 - s(t)]$, $[1, +\infty)$ we have

$$\left.\begin{aligned} \operatorname{Im}\{f_1(z;t)\} &= 0, \\ \operatorname{Re}\{f_2(z;t)\} &= 0. \end{aligned}\right| \qquad (12)$$

It also results from Eq. (10) that:

$$\left.\begin{aligned} &\operatorname{Im}\left\{\frac{\partial f_1(z;t)}{\partial t}\right\} = 0, \\ &\qquad\qquad\qquad \text{for } z = x \in (1 - s(t), 1). \\ &\operatorname{Re}\left\{\frac{\partial f_2(z;t)}{\partial t}\right\} = 0, \end{aligned}\right| \qquad (13)$$

The expressions lead to the conditions: /542

$$\left.\begin{aligned} &\operatorname{Im}\{f_1(z;t)\}_{y\to+0} = U_1(x), \\ &\qquad\qquad\qquad x \in (1 - s(t); 1), \\ &\operatorname{Re}\{f_2(z;t)\}_{y\to+0} = U_2(x), \end{aligned}\right| \qquad (14)$$

$U_1(x)$ and $U_2(x)$ are momentarily arbitrary functions. Likewise, using the above notations, the limiting conditions (5) assume the form:

$$\left.\begin{aligned} &\operatorname{Im}\{f_1(z;t) + f_2(z;t)\}_{y\to+0} = -\Psi_+(x;t), \\ &\qquad\qquad\qquad -1 - s < x < 1 - s. \\ &\operatorname{Im}\{f_1(z;t) + f_2(z;t)\}_{y\to-0} = -\Psi_-(x;t), \end{aligned}\right| \qquad (15)$$

(We have assumed that the airfoil profile starts at rest with $t = 0$. The general case should be similarly treated.)

Taking into account Eqs. (12) and (14), the following limiting conditions are obtained from these expressions in order to determine the functions $f_1(z;t)$ and $f_2(z;t)$:

$$
\begin{aligned}
&\mathrm{Im}\,\{f_1(z;t)\}_{y=0} = 0, \qquad x \in (-\infty; -1-s) \cup (1; +\infty),\\
&\mathrm{Im}\,\{f_1(z;t)\}_{y=+0} = U_1(x), \qquad x \in (1-s; 1),\\
&\mathrm{Im}\,\{f_1(z;t)\}_{y=+0} = -[\dot{Y}_+(x;t) - \dot{Y}_-(x;t)], \qquad x \in (-1-s; 1-s),
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\mathrm{Re}\,\{f_2(z;t)\}_{y=0} = 0, \qquad x \in (-\infty, -1-s) \cup (1; +\infty),\\
&\mathrm{Re}\,\{f_2(z;t)\}_{y=+0} = U_2(x), \qquad x \in (1-s; 1),\\
&\mathrm{Im}\,\{f_2(z;t)\}_{y=+0} = -[\dot{Y}_+(x;t) + \dot{Y}_-(x;t)], \quad x \in (-1-s, 1-s).
\end{aligned}
\tag{17}
$$

Function $f_1(z;t)$ is thus a solution for a Dirichlet problem for the upper semi-plane:

$$
f_1(z;t) = \frac{\varepsilon}{\pi}\int_{-1-s(t)}^{1-s(t)} \frac{\dot{Y}_+(\xi;t) - \dot{Y}_-(\xi;t)}{z-\xi}\,d\xi - \frac{1}{\pi}\int_{1-s(t)}^{1} \frac{U_1(\xi)}{z-\xi}\,d\xi. \tag{18}
$$

Function $U_1(\xi)$ will be determined on condition that function $f_1(t;z)$ behaves at infinity like z^{-2}, which leads to the integral equation:

$$
\int_{1-s(t)}^{1} U_1(\xi)\,d\xi = \varepsilon\int_{-1-s(t)}^{1-s(t)} [\dot{Y}_+(\xi;t) - \dot{Y}_-(\xi;t)]\,d\xi, \tag{19}
$$

but

$$
\dot{Y}_\pm(x;t) = \frac{\partial Y_\pm(x;t)}{\partial t} = \frac{\partial \widetilde{Y}_\pm(x+s,t)}{\partial x} V_0(t) + \frac{\partial \widetilde{Y}_\pm(x+s,t)}{\partial t}.
$$

In (19) we shall make a change of variable $\xi + s = \xi_1$, /543

$$
\begin{aligned}
&\int_{1}^{1+s} U_1(\xi_1 - s)\,d\xi_1 =\\
&= \varepsilon\int_{-1}^{+1}\left\{\frac{\partial[\widetilde{Y}_+(\xi_1;t) - \widetilde{Y}_-(\xi_1;t)]}{\partial \xi_1} V_0(t) + \frac{\partial[\widetilde{Y}_+(\xi_1;t) - \widetilde{Y}_-(\xi_1;t)]}{\partial t}\right\} d\xi_1
\end{aligned}
\tag{20}
$$

or

$$\int_{1}^{1+s} U_1(\xi_1 - s)\, d\xi_1 = \frac{d}{dt}\, \varepsilon \int_{-1}^{+1} [\widetilde{Y}_+(\xi;t) - \widetilde{Y}_-(\xi,t)]\, d\xi \equiv \dot{a}(s) \tag{20'}$$

where we designated the area of the airfoil by a(s).

From expression (21) we obtain by derivation

$$U_1(1 - s) = \frac{d\dot{a}(s)}{ds}. \tag{21}$$

We shall note that function $U_1(x)$ is different from zero only if the area of the airfoil changes during the motion. Physically, this describes the intensity of a source situated on the segment (1 - s, 1) on axis Ox. Once we have determined the function $f_1(z;t)$ from expression [3]:

$$\lim_{y \to \pm 0} \frac{\partial f_1(z;t)}{\partial t} = \pm i \frac{\varepsilon}{2} \frac{\partial l(x;t)}{\partial x} + \frac{\varepsilon}{2\pi} \int_{-1-s}^{1-s_0} \frac{l(\xi_1;t)}{(x-\xi)^2} d\xi$$

Function $l(x;t)$ is also obtained (taking into account Kutta-Jukowsky's condition).

The problem under limiting conditions for determining the function $f_2(z;t)$ is identical to the limiting problem which determines the unsteady motion of the thin airfoil without thickness, according to W. Birnbaum's hypothesis. Its solution is given, for example, in [1], and it leads to the integral of H. Wagner's equation for determining the function $U_2(x)$. From the expression

$$\lim_{y \to \pm 0} \frac{\partial f_2(z;t)}{\partial t} = \mp \frac{1}{2} \frac{\partial n(x;t)}{\partial x} + \frac{i}{2\pi} \int_{-1-s}^{1-s_0} \frac{n(\xi_1;t)}{(x-\xi)^2} d\xi$$

function n(x;t) is also obtained.

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